Spatiotemporal Chaos

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The term "chaos" denotes persistent irregular behavior of a deterministic system (that is, one in which externally applied noise can be neglected). Much of the work on chaos of the last 15 years has dealt with systems that could be represented by a small number of degrees of freedom, such as the logistic map $u_{t+1} = ru_t(1 - u_t)$ or the famous Lorenz model involving nonlinear differential equations for three coupled dynamical variables. Remarkably, at least near enough to the threshold of chaos, continuous experimental systems such as stirred chemical reactors or flows in certain fluid cells can be represented by such models.

Methods have been developed for analyzing chaotic behavior that are well adapted to these systems, such as measuring Lyapunov exponents and fractal dimensions of strange attractors by various "reconstruction techniques" (*1*). Recently, however, chaotic systems have been studied that are not reducible to a model with a small number of degrees of freedom, even at the onset of chaos. Such systems are said to be "large" and to display "spatiotemporal chaos," because their description appears to require a large number of chaotic elements distributed in space.

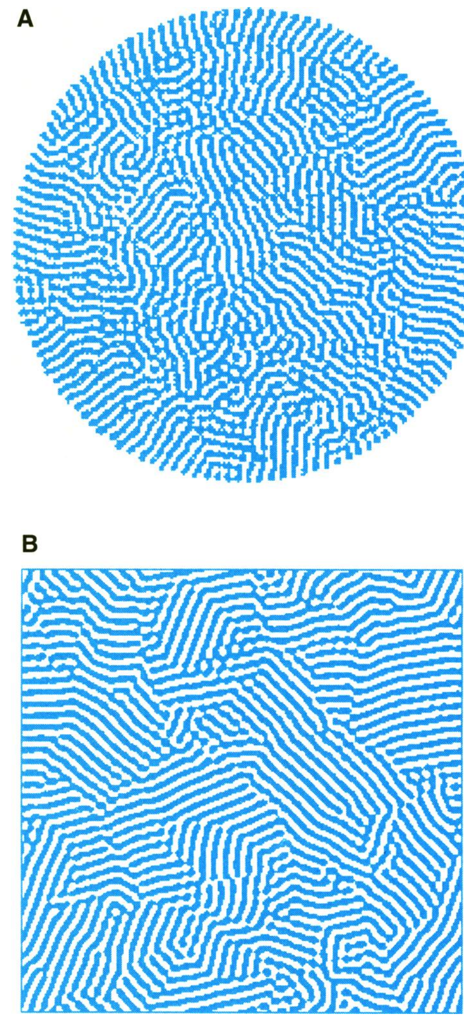
An exciting advance in the study of spatiotemporal chaos is the development of a number of experimental systems that are well characterized and precisely controlled, and in some cases approach the ideal of large system size so that a statistically homogeneous state, independent of boundary effects, seems to exist over much of the system. Such examples offer the real possibility of understanding spatiotemporal chaos by means of a combination of theoretical, numerical, and experimental techniques that have been successfully applied to the study of regular spatial patterns in nonequilibrium systems (*2*).

The rotating convection system is a representative example of recent work. The nonrotating case, the familiar Rayleigh-Bénard convection, has served well as a canonical system for the study of time-independent spatial patterns and their transients. It is experimentally simple—a fluid held between two precisely horizontal plates with the lower plate carefully maintained at a higher temperature than the upper plate—and well described theoretically by the equations of fluid dynamics for a viscous fluid driven by the buoyancy force in a gravitational field.

If a Rayleigh-Bénard system is rotated at an angular frequency Ω about the vertical, then there exists a critical rotation rate Ω_c such that for $\Omega < \Omega_c$, the usual ordered time-independent state is stable, whereas for $\Omega > \Omega_c$, a chaotic state is found even at the threshold of the spatial pattern (*3*). This spatiotemporal state takes the appearance of a system of interacting domains with finite lifetimes (see figure). The domain size may be used to define a characteristic correlation length (*4*) of the system: Both the lifetime and this correlation length depend on the experimental parameters. A numerically tractable model, based on previous experience in understanding stationary patterns and known as a "generalized Swift-Hohenberg equation" (*5*), has been used to model the system (*6*). In addition, the proximity to threshold suggests that a weakly nonlinear theory can lead to a simpler model which provides some analytic understanding of the behavior (such as the dependence of the correlation length on the distance to threshold) (*7*). An interesting prediction of this work, confirmed by the numerical modeling, is the bistability of the system—for the same parameters, the state may either be a time-independent one of ordered rolls or a disordered chaotic state, depending on how the system is prepared.

Other examples of spatiotemporal chaos have been studied in recent years in systems that allow different levels of quantitative characterization and statistical reproducibility and approach the ideal of a "large" system to various degrees. A particularly dramatic example is a state discovered in the standard Rayleigh-Bénard system, when for some parameter values an ordered state of straight or weakly curved rolls breaks down to a spatiotemporally chaotic state consisting of elementary spiral structures which appear and disappear in an irregular fashion (*8*). This chaotic state has also been successfully modeled with an appropriate generalized Swift-Hohenberg equation (*9*). Other examples include various disordered states in convection systems (*10*), electrohydrodynamic convection in nematic liquid crystals (*11*), various pattern-forming chemical reactions (*12*), convection in binary fluid mixtures (*13*), and parametrically excited surface wave patterns in fluids (*14*).

A number of theoretical questions are raised by these experiments: For instance, what broad classes of behavior exist in spatiotemporal systems? We are immediately struck by questions familiar in equilibrium statistical mechanics: Can different phases, separated by sharp transition points, be identified within the chaotic state? How much, if any, of our understanding of the effect of thermal fluctuations on equilibrium phases carries over to an understanding of the effect of chaotic fluctuations on nonequilibrium phases? Are there any universal features at transitions to chaotic states or at transitions between different chaotic states, and if so, how do the univer-



Chaotic domain state: (**A**) in experiments on rotating convection in CO_2 at 20 atmospheres in a cylindrical cell (*3*), with hot and cold fluid visualized with a digitally enhanced shadowgraph; and (**B**) from numerical simulation of a generalized Swift-Hohenberg model (*6*), with positive values of the field shown black and negative values white. Both patterns continue to evolve in an irregular way. The similarity between the numerical results and the portion of the experimental cell away from the boundaries suggests a boundary-independent spatiotemporally chaotic state.

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sality classes relate to the known ones in equilibrium systems? In addition, there are also likely to be other, yet to be formulated questions that are specific to the nonequilibrium nature of the systems.

What are the appropriate qualitative measures of chaotic behavior? Is there a compact way of defining spatiotemporal chaos? And what quantitative measures are useful? The conventional characterization techniques for chaos with a few degrees of freedom, such as Lyapunov exponents and fractal attractor dimensions (*1*), involve the geometry of the motion in the full phase space of the system. For a spatially distributed system, the phase space is enormous, and it is not clear whether the traditional diagnostics can be suitably modified. One would have to seek extensive quantities to define intensive densities, which may become independent of the system size for large enough systems. To be practical, there must be ways to calculate appropriate quantitative measures by studying small subsystems of the spatially extended system.

It is natural in our present stage of understanding to attempt to characterize the systems in terms of correlation times and lengths. The relation between correlation times or lengths defined through different properties (for example, from the two-point correlation function or from the attractor dimension) is unknown and could be quite complicated. However, if there is a divergence approaching some point in parameter space, a comparison between the divergences of different lengths or times becomes particularly interesting.

Are there simple reduced descriptions, emphasizing the collective behavior of many chaotic degrees of freedom, again analogous to the reduced long-wavelength descriptions provided by thermodynamics and hydrodynamics for equilibrium systems? Are there simple limits that can be studied? One possibly useful limit is weak coupling. An example is a set of mappings (such as the logistic map) placed on lattice sites, with the dynamics of each map weakly (through a coupling parameter g) dependent on the dynamics of its neighbors in some specified way. For $g = 0$ the behavior is completely understood as the sum of the individual map properties, and the system is clearly extensive. For example, if the dimension of the attractor of the single map is d_f, then the dimension of N maps on a lattice is $D_f = Nd_f$ in this limit. It is natural to assume that for weak coupling ($g << 1$), such results continue to apply approximately, so that we expect $D_f = N\rho$, where ρ is a *dimension density* of the form $\rho = d_f + 0(g)$. For general values of the coupling, we still expect the dimension D_f to be extensive, but the dependence of the dimension density ρ on the coupling g is more complicated.

Are there analytically tractable theoretical models of spatiotemporal chaos? In this connection, the work of Hansel and Sompolinsky (*15*) should be mentioned, where a lattice model of coupled m-component elements is solved exactly in the limit as m goes to infinity and is reduced to a single degree of freedom in the presence of a Gaussian noise, which must then be determined self-consistently.

Weak spatiotemporal chaos is a ubiquitous phenomenon in large nonequilibrium systems near the threshold to pattern formation. Recent experimental and theoretical work has identified a number of well-characterized systems showing this behavior and yielding detailed data on the spatiotemporal evolution. What is lacking is a simple phenomenology of spatiotemporal chaos that would reveal the essential features buried in the wealth of available data. Developing such an understanding is a challenge to theorists and experimentalists in the field of nonequilibrium phenomena.

References and Notes

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Time *Is* the Essence: Molecular Analysis of the Biological Clock

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Biological clocks underlie rhythmic processes from daily variation in photosynthesis in the single-celled dinoflagellates to the annual breeding cycles of some mammals. By the 1970s, it was clear that such clocks were ubiquitous and that their formal properties were similar among phylogenetically diverse organisms. The stage was set for tackling one of the major questions: What is the mechanism responsible for the generation of circadian (24-hour) oscillations?

The early demonstration of circadian rhythms in unicellular organisms had pointed to intracellular, biochemical processes as the underlying mechanism. The paradigm illustrated in the upper part of the figure has formed the heuristic basis for the experimental attack on mechanism. The oscillator is a negative feedback *loop* in which individual elements function either as *state variables* (A–D) of the oscillator or as *parameters* (*a*–*d*), which mediate their interaction. Pathways for input to the loop and for output complete the model. The biochemical problem was to identify the molecular correlates of the input, loop, and output elements.

One approach to the problem was genetic. Could a mutational analysis lead to the identification of genetic loci and the products of their expression that were part of the central mechanism? The isolation of single gene mutations that had profound effects on the circadian phenotype initially generated much excitement. In particular, in the early 1970s Konopka (*1*) and Feldman (*2*) discovered the *per* locus in *Drosophila melanogaster* and the *frq* locus in *Neurospora crassa*, both in organisms in which the power of genetics was well established. These discoveries greatly raised the expectations of the field. Mutations in both *per* and *frq* either abolished rhythm expression or altered its period, suggesting that these loci were central to clock function. But in the ensuing decade little new information on the biochemical function of these loci was forthcoming, and the genetic analysis seemed less productive than had

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